The Free Coating Process and Its Relationship to Drainage

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For steady state drainage of a laminar film down a vertical plate (*2*), the volumetric flow rate per unit plate width is given by Nusselt's expression as $\Gamma = (\rho g h^3/3\mu)$. The experimenter controls the flow rate Γ, hence the film thickness h is uniquely determined. Consider now the continuous coating of an infinite plate by withdrawal from a liquid bath at a constant velocity u_o. If no obstruction is applied to the liquid film formed (such as a doctoring blade), the process is referred to in the coating industries as *free coating*.

For thin films the flow equation for drainage or withdrawal in the region where surface tension has no effect reduces to

$$0 = \rho g_x + \mu \frac{\partial^2 u}{\partial y^2} \quad (1)$$

where g_x is $+g$ when the x coordinate is directed downward as in the case of drainage or $-g$ when x is directed upward as in the case of withdrawal. The coordinates of the system are shown in Figure 1.

In Equation (1) the inertial terms such as $\rho\ \partial u/\partial t$ were neglected under the assumption that they are small as compared with viscous and gravitational terms. As shown recently (*3*) this is a very good assumption for most practical cases.

Integration of Equation (1) for the withdrawal case under the conditions of no slip on the plate and no shear at the air-liquid interface results in the known relationship between film thickness and flow rate.

$$\Gamma = u_o h - \frac{\rho g h^3}{3\mu} \quad (2)$$

Contrary to the drainage case, however, the withdrawal problem is not uniquely determined at this stage of the solution, because in the withdrawal case, the experimenter sets only u_o but has no control over the flow rate Γ. Thus, there are two unknowns remaining, Γ and h, and only one equation relating them. To obtain a closed solution an additional constraint is needed. This constraint was supplied by Levich and Landau (*6, 7*), who considered the surface tension effects near the liquid bath. They assumed that the liquid was being squeezed out from under the static meniscus and obtained a solution for the film thickness as a function of the withdrawal speed and fluid properties including surface tension. An extension of this theory has been recently provided by White and Tallmadge (*9*). Levich (*7*) was aware of the shortcomings of his theory, because he knew that the assumption of the static meniscus holds, at best, for very slow withdrawals. Moreover, experiments showed that at fast withdrawals surface tension had no effect on film thickness (*1*). Thus, he advanced a new theory (*7*) based on dimensional analysis, resulting in the following relation for the film thickness.

$$h = A\sqrt{\frac{\mu u_o}{\rho g}} \quad (3)$$

The free constant A was determined experimentally to equal unity.

Deryagin (*1*) used a different approach to solve this problem by optimizing the flow rate as given by Equation (2). The maximum flow rate is obtained when $h = \sqrt{\mu u_o/\rho g}$. However, it is not evident a priori that the flow rate should be a maximum.

This communication examines the relationship between drainage of a liquid from a plate and withdrawal of a plate from a bath. By doing this, the solution for the film thickness as given by Deryagin (*1*) and Levich (*7*) is obtained on rational grounds. It is also explained why the steady state flow rate in withdrawals where surface tension has no effect is a maximum. In addition the solution for the unsteady state withdrawal, so far unpublished, is presented.

The free coating or withdrawal problem is related to drainage by a simple coordinate transformation. Consider the problem of nonsteady state drainage from a

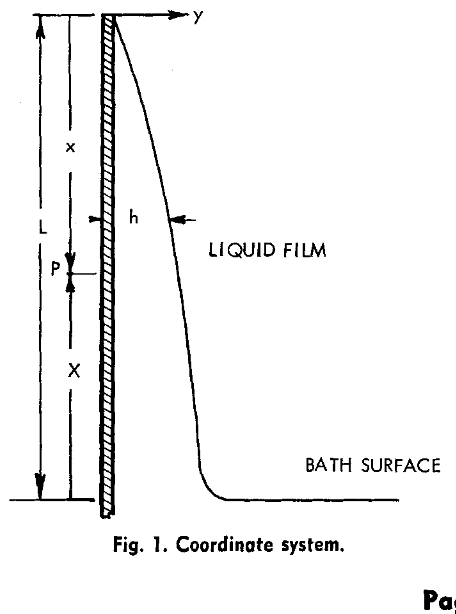

Fig. 1. Coordinate system.

flat plate. Jeffreys (5) has shown that the film thickness is given by

$$h = \sqrt{\frac{\mu x}{\rho g t}} \qquad (4)$$

where x is the distance as measured from the upper edge of the wetted portion of the plate. Assume that the plate is being withdrawn from a bath at velocity u_o. Let L be the length of the wetted portion of the plate withdrawn. The relationship between L, u_o, and elapsed time t is

$$L = u_o t \qquad (5)$$

Since the liquid on the plate does not know whether the plate is stationary or moving with velocity u_o, the film thickness h is still given by Equation (4) for any distance x measured from the top ($x < L$).

Now let P be some fixed point in space a distance X above the bath surface. The distance of this point P from the upper edge of the wetted portion of the moving plate is some value of x.

The total length L of the wetted portion of the plate is (see Figure 1)

$$L = u_o t = x + X \qquad (6)$$

which may be written as

$$x = u_o t - X \qquad (7)$$

Now to an observer at point P located some fixed distance X above the bath surface the film thickness h is given by Equation (4). The distance x is a function of t and X, given by Equation (7). Substitution of (7) into (4) gives

$$h = \sqrt{\frac{\mu}{\rho g}(u_o - X/t)} \qquad (8)$$

which is now the nonsteady state film thickness for the withdrawal problem. Equation (8) is of course subject to the restrictions that X be large enough so that meniscus effects are negligible and that $X/t < u_o$. It is also assumed that the dimensionless parameter $u_o\mu/\sigma$ is large enough so that surface tension no longer affects film thickness at some distance far enough above the bath level (*1*). As $t \to \infty$ we get the solution for the steady state thickness h_o:

$$h_o = \sqrt{\frac{\mu u_o}{\rho g}} \qquad (9)$$

From Equation (8) it is seen that for a given X, the maximum value that h can have is the steady state value of Equation (9). Thus, as H increases from zero to $\sqrt{\mu u_o/\rho g}$, the flow rate Γ increases from zero to its maximum and final value

$$\Gamma_{max} = \frac{2}{3} u_o \sqrt{\frac{\mu u_o}{\rho g}} \qquad (10)$$

Close to steady state the term X/t will be small; thus, in this region Equation (8) can be approximated by

$$h = \sqrt{\frac{\mu u_o}{\rho g}}\left(1 - \frac{1}{2}\frac{X}{u_o t}\right) \qquad (11)$$

From (11) one obtains the time required to reach a thickness at a distance X above the bath differing from h_o by 1%:

$$t = 50\frac{X}{u_o} \qquad (12)$$

As seen from Equation (12) this time is certainly not negligible, which may explain why Van Rossum's data (8) for finite plate withdrawal result in 10% lower thicknesses than those predicted by Equation (9).

Everything said above can, of course, be easily extended to any non-Newtonian fluid. Thus, for instance, the unsteady state withdrawal from a power law fluid can be easily adopted from the drainage solution given in the literature (*4*):

$$h = \left[\frac{K}{\rho g}(u_o - X/t)^n\right]^{\frac{1}{n+1}} \qquad (13)$$

The power law equivalents of Equations (11) and (12) will be, respectively

$$h = \left[\frac{K u_o}{\rho g}\right]^{\frac{1}{n+1}}\left(1 - \frac{n}{n+1}\frac{X}{u_o t}\right) \qquad (14)$$

and

$$t = \frac{100\,n}{n+1}\cdot\frac{X}{u_o} \qquad (15)$$

Comparing Equations (12) and (15) we note that for pseudoplastic fluids where $n < 1$, the time required to reach steady state is shorter than that for Newtonian ones.

It has been previously recognized (*4*) that the solution for film thickness in the steady state withdrawal problem could be obtained from the nonsteady state drainage problem by substituting u_o for x/t, but it is believed that this is the first time that the explicit relatoinship has been set forth.

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NOTATION

- g = gravitational acceleration
- h = film thickness
- h_o = film thickness at steady state
- K = rheological constant, consistency index
- n = rheological constant, power law exponent
- t = time
- u = velocity in the plate direction
- u_o = velocity of solid support
- x = coordinate, distance from the top of draining plate
- X = coordinate, distance from the bath level
- y = coordinate normal to the film

Greek Letters

- Γ = volumetric flow rate per unit plate width
- μ = viscosity (Newtonian)
- ρ = density
- σ = surface tension at the liquid-air interface

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